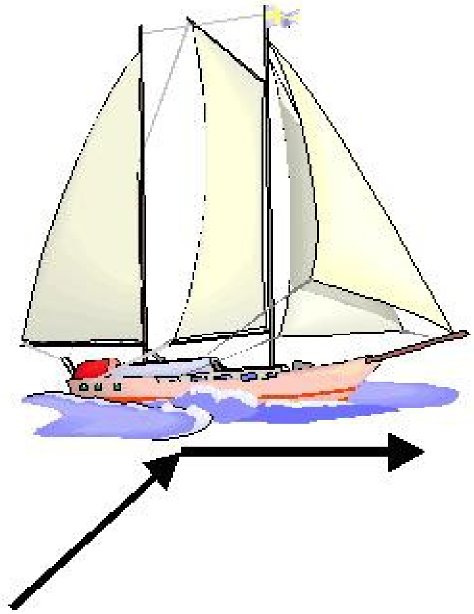
Activity: Vector Voyage!



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Summary

In this activity, students will use vector analysis to understand the concept of dead reckoning. Students will use vectors to plot their course based on a time and speed. They will then correct the positions with vectors representing winds and currents.

Engineering Connection

Relating math concept to engineering

Although first described by mathematicians, nearly every branch of engineering uses vectors as a tool today, especially to calculate force and stress. Mechanical, aerospace, civil and chemical engineers who design using fluid dynamics concepts use vectors in their calculations to describe real-world forces such as wind and water movement. Electrical engineers also use them to describe the forces of magnetic and electric fields.

Contents

Grade Level: 8 (7-9)

Time Required: 35 minutes

Group Size: 1

Activity Dependency :None

Expendable Cost Per Group : US$ 0
(assumes students have colored pencils; excludes cost for printing worksheets)

Keywords: dead reckoning, distance, direction, navigation, Pythagorean Theorem, vectors

Related Curriculum :

subject areas	Earth and Space Geometry Measurement
curricular units	Plot Your Course - Navigation
lessons	How to be a Great Navigator!

Educational Standards

- Colorado: Math
- b. Use representations of linear functions to analyze situations and solve problems (Grade 8) [2009]
- b. Use the Pythagorean Theorem to find unknown lengths in right triangles (Grade 8) [2009]
- Colorado: Science
- b. Describe methods and equipment used to explore the solar system and beyond (Grade 8) [2009]
- Common Core State Standards for Mathematics: Math
- 1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). (Grades 9 - 12) [2010]
- 3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. (Grades 9 - 12) [2010]
- 4. (+) Add and subtract vectors. (Grades 9 - 12) [2010]
- a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (Grades 9 - 12) [2010]
- International Technology and Engineering Educators Association: Technology
- F. Knowledge gained from other fields of study has a direct effect on the development of technological products and systems. (Grades 6 - 8) [2000]

Learning Objectives (Return to Contents)

After this activity, students should be able to:

- Understand that vectors can represent distances and directions and are a good way to keep track of movement on maps.
- Use vectors to understand directions, distances, and times associated with movement and speed.
- Understand how vector analysis is used in nearly every branch of engineering

Materials List

Each student should have:

- 1 each Vector Voyage Worksheets 1 and 2
- 3 different colored pencils (blue, green and red match Worksheet instructions)

Introduction/Motivation (Return to Contents)

Can you describe speed and distance? (Answer: distance = speed x time; it is a good idea to write this on the board. Remind students that the units have to match. For example, if the speed is measured in miles per hour, then the time must be converted into hours for the formula to work correctly.) How did ancient sea captains keep their ship on course throughout their voyage? (Answer: They used *dead reckoning* to figure out where they were going.) Did they follow the sun, the shoreline or even the stars? (Answer: Yes. However, by knowing the speed, time and course of their travel, they could determine where and approximately when they would arrive.) Columbus — and most other sailors of his era — used dead reckoning to navigate. With dead reckoning, the navigator finds their position by estimating the course and distance they have sailed from some known point. Starting from a known point, such as a port, the navigator measures out their course and distance from that point on a chart, pricking the chart with a pin to mark the new position. These early navigators used math to help them find their way and stay on course when wind, current and other factors might affect their journey. Unfortunately, Columbus never actually reached the destination where he thought he would end up. Why do you think that happened? How accurate is dead reckoning?

Procedure

Dead reckoning is the process of navigation by advancing a known position using course, speed, time and distance to be traveled. In other words, figuring out where you will be at a certain time if you hold the *speed, time* and *course* you plan to travel.

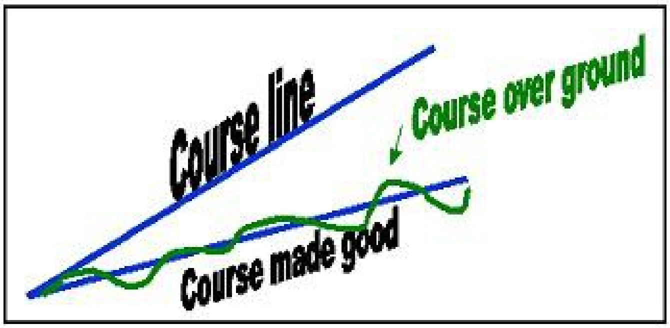

Figure 1. Graphical illustration of a vessel's voyage using vectors.
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The course is the direction you intend to steer the vessel. For this exercise, the "course," or heading, is always due west (270 degrees measured clockwise from 0 degrees north). A heading is which way the vessel is going at a given point. The track actually followed can be very crooked due to wave action, current, wind and the helmsman (the person responsible for steering the vessel). Course made good is the course actually traveled.

Vectors are arrows that represent two pieces of information: a magnitude value (the length of the arrow) and a directional value (the way the arrow is pointed). In terms of movement, the information contained in the vector is the distance traveled and the direction traveled. Vectors give us a graphical method to calculate the sum of several simultaneous movements. If movement is affected by only one variable (represented by vector A or B), then a vessel would arrive at the end of that vector. If movement is affected by two variables (represented by the sum of A and B), then a vessel's final position can be found by linking the two vectors together.

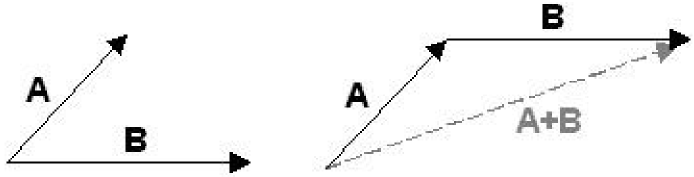

Figure 2. Vectors illustrate the final position of vessel's voyage.
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Before the Activity

- Print a copy of the Vector Voyage Worksheets 1 and 2 for each student.
- Print Vector Voyage Solution Sheet 1, 2 and 3 for yourself as answer keys.
- Provide a brief introduction on vectors to the students.

With the Students

Ask the students: Should sailors worry about wind and current when traveling long distances? (Answer: Yes. Wind and currents can take a ship far from the course it would follow otherwise. If the navigator is not keeping track of the affects of the wind and current, the ship could become hopelessly lost.)
Ask the students: How are vectors related to speed? (Answer: A (velocity) vector will tell both speed and direction (N, S, E, W), while speed alone does not tell you direction.)

1. Give each student a Vector Voyage Worksheet 1.
2. Using the specified color of pencil, have them draw the 10 square movement vectors straight across the map and answer the questions on the worksheet.
3. Have the students redraw the 10 square movement vectors on the map while adding the wind vector corrections for each month. Each month's movement vector must start from the end of the previous month's wind vector (refer to Vector Voyage Solution Worksheet 1). Have the students answer the questions on the worksheet.
4. Have the students redraw the 10 square movement vectors *and* wind correction vectors on the map while adding the current vector corrections for each month. Each month's current vector now starts from the end of the previous month's wind vector. Each month's movement vector must now start from the end of the previous month's current vector (refer to Vector Voyage Solution Worksheet 2). Have the students answer the questions on Vector Voyage Worksheet 2.
5. Once they are done, point out how they would have landed on the U.S. without the effects of wind or ocean currents. However, because of wind and ocean currents, they ended up in Cuba.
6. Tell the students that each square is 100 miles in length. Have them calculate the distance for Part 1. (Answer: 3,500 miles.)

Attachments (Return to Contents)

- Vector Voyage Worksheet 1
- Vector Voyage Worksheet 2
- Blank Vector Voyage Worksheet
- Vector Voyage Solution Worksheet 1
- Vector Voyage Solution Worksheet 2
- Vector Voyage Solution Worksheet 3

Troubleshooting Tips

Getting started drawing the vectors may be confusing for students. If necessary, help the students by drawing the first two vectors with them on the chalkboard or in groups.
The wind correction vector is added to the end of the first vector arrow for month 1. The vectors for Part 3 of the worksheets must build off of the added vectors in Part 2. Both the wind and the ocean affect the landfall; this is represented accurately only by building off the wind correction vectors.
Vector Voyage Solution Worksheet 3 offers a summary of this activity and clearly illustrates the vector movement directly. This solution worksheet is an excellent teacher reference for students who are having difficulty with this exercise.
Assessment (Return to Contents)

Pre-Activity Assessment

Discussion Question: Solicit, integrate and summarize student responses.

- Should sailors worry about wind and current when traveling long distances? (Answer: Yes. Wind and currents can take a ship far from the course it would otherwise follow.

- Should a navigator pay attention to wind? To current? (Answer: Yes. If the navigator is not keeping track of the affects of the wind and current, the ship could become hopelessly lost.)

Activity Embedded Assessment

Worksheet: Have the students complete the activity worksheet; review their answers to gauge their mastery of the subject.

- Have students fill out the questions on the Vector Voyage Worksheets (as directed in "With the Students" of the Procedure section).

Post-Activity Assessment

Student Generated Question: Solicit, integrate and summarize student responses.

- Have each student pick a spot on the African coast and then determine the wind and current correction vectors that would take their ship there after 1 month of sailing east 10 squares. They should exchange these corrections with a partner (without letting the partner see their sheet), and calculate where they would arrive in Africa using their partner's corrections on their own sheet.

Activity Extensions (Return to Contents)

Using blank Vector Voyage Worksheets, have students plot their own courses — recording movements, directions and corrections along the way. They should give the new course instructions to a partner to determine if s/he can sail to the new spot.
Activity Scaling

- For 6th grade, do the wind correction part of the activity together as a class. Have the students try the current correction on their own.
- For 7th grade, do the activity as is.
- For 8th grade, have the students calculate the actual total distance traveled by the ship on the way to Cuba. The actual distance traveled by the ship is the resulting vector from the sum of the three movement vectors each month. Students can draw these vectors on their maps by starting at the beginning of the solid 10 square vector for each month and drawing an arrow straight to the final position of the ship for that month. Use the Pythagorean Theorem ($a^2+b^2=c^2$) to find the lengths of these vectors. (Answer: The distance from Spain to Cuba is 3,683 miles.) The students should also be able to calculate the distance from Spain to Florida in the same manner. (Answer: 3,940 miles)
- For 8th grade (if more is needed), have the students calculate the speed of the ship in miles per month and miles per hour. (Rate=distance/time) (Answer: Florida is 1.37 mph or 985 miles/month, Cuba is 1.7 mph or 1,228 miles/month, and New York is 1.39 mph or 1,000 miles/month.) Are these speeds fast or slow? How about for a time with no engines? What would happen to the food supply if there were always a breeze of 6 squares east?

Contributors

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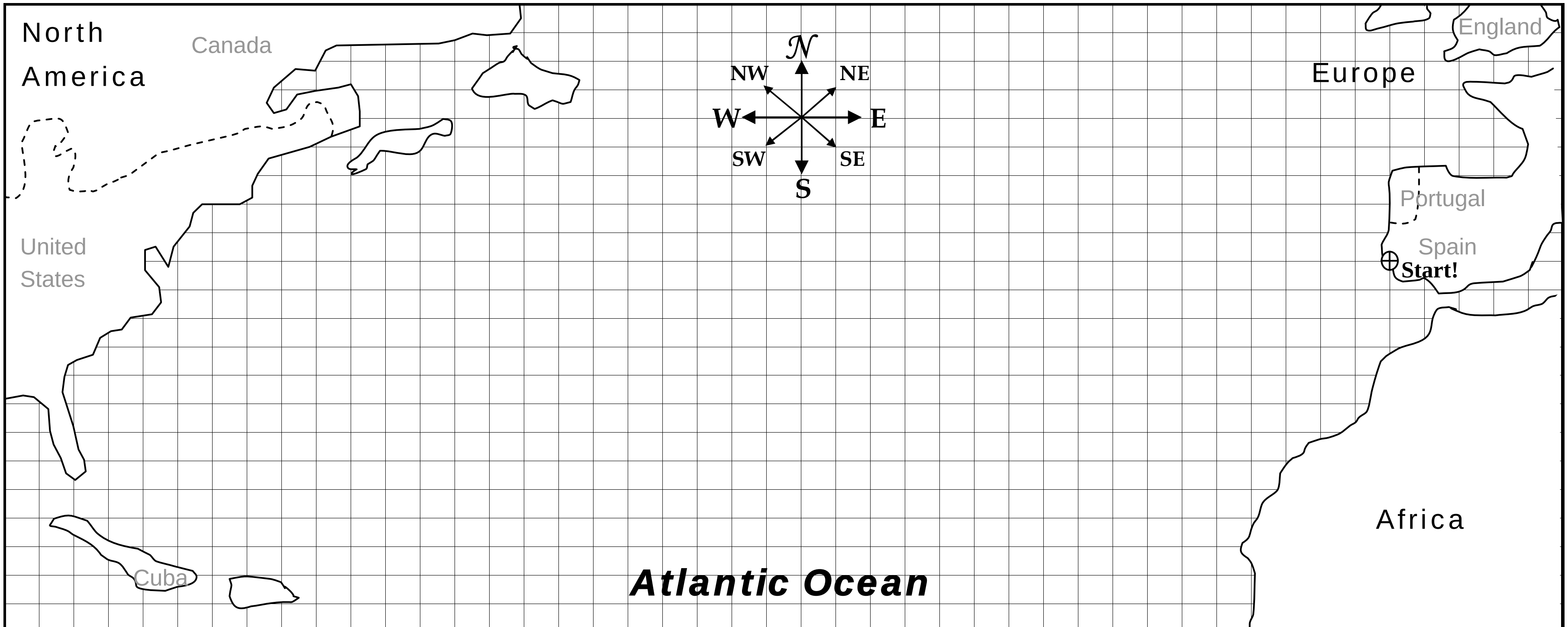
Supporting Program (Return to Contents)

Integrated Teaching and Learning Program, College of Engineering, University of Colorado at Boulder
Last Modified: July 5, 2012

Name: ____ **Date:** ____________________

Vector Voyage Worksheet 1



Vector Voyage Instructions

Part 1: Your ship can sail 10 squares/month. Starting from Spain and travelling west (or bearing 270), draw one vector for each month of travel using your red pencil. In what country will you make landfall? ___________ How many months will it take to reach land (how many 10-square vectors is it)?___________

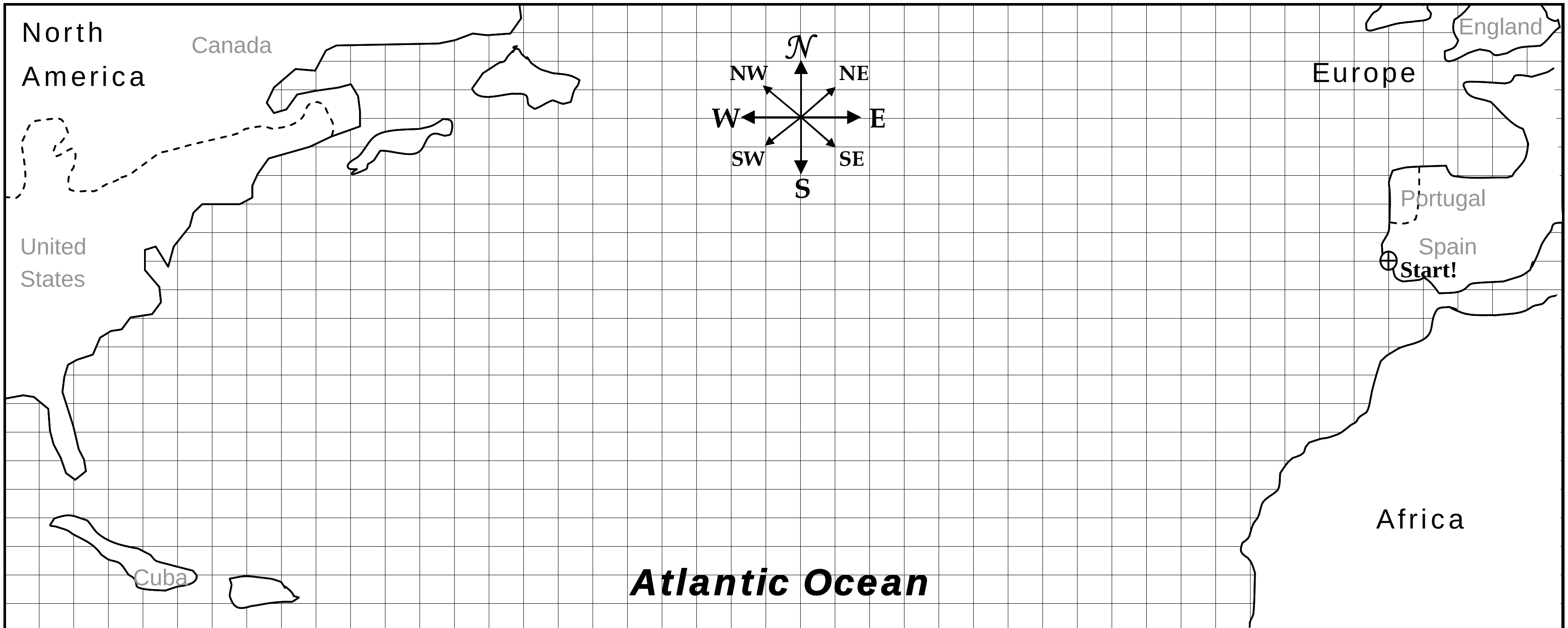
Part 2: Unfortunately, the wind does not always blow the way you want! To determine how the wind effects our travel, let's include the wind vector. First, draw your ship vector, just like in part 1, using your red pencil. Now at the end of that vector, add the wind vector using your blue pencil. Now draw the resulting vector (just adding the two vectors) with your blue pencil. That's where you are when the first month ends. Now do the same for the next month and the month after that until you reach land. Remember that the wind changes, so each month you will have to add a different wind vector. The list of different winds for each month is on the following line.

Month 1: 2 squares south Month 2: 4 diagonal squares southeast Month 3: 2 squares west Month 4: No wind

Where will you make landfall now? ___________ How many months to reach land (only count the solid 10-square vectors!)?___________

Name: ____ **Date:** __________________

Vector Voyage Worksheet 2



Vector Voyage #2 Instructions

Part 3: Unfortunately, ocean currents affect boats too! Each month, you must also add a current correction vector to find your actual final position. So just like in Part 2, for each month, draw your ship vector (in red), and your wind vector (in blue) and now add on your ocean vector (in green). Now using your green pencil, draw the resulting vector from all of these (add the red and the blue and green vectors). Do this for each month until you hit land. Remember the wind and the ocean vector's will be different for each month. They are listed below.

Wind Vectors:

Month 1: 2 squares south Month 2: 4 diagonal squares southeast Month 3: 2 squares west Month 4: No wind

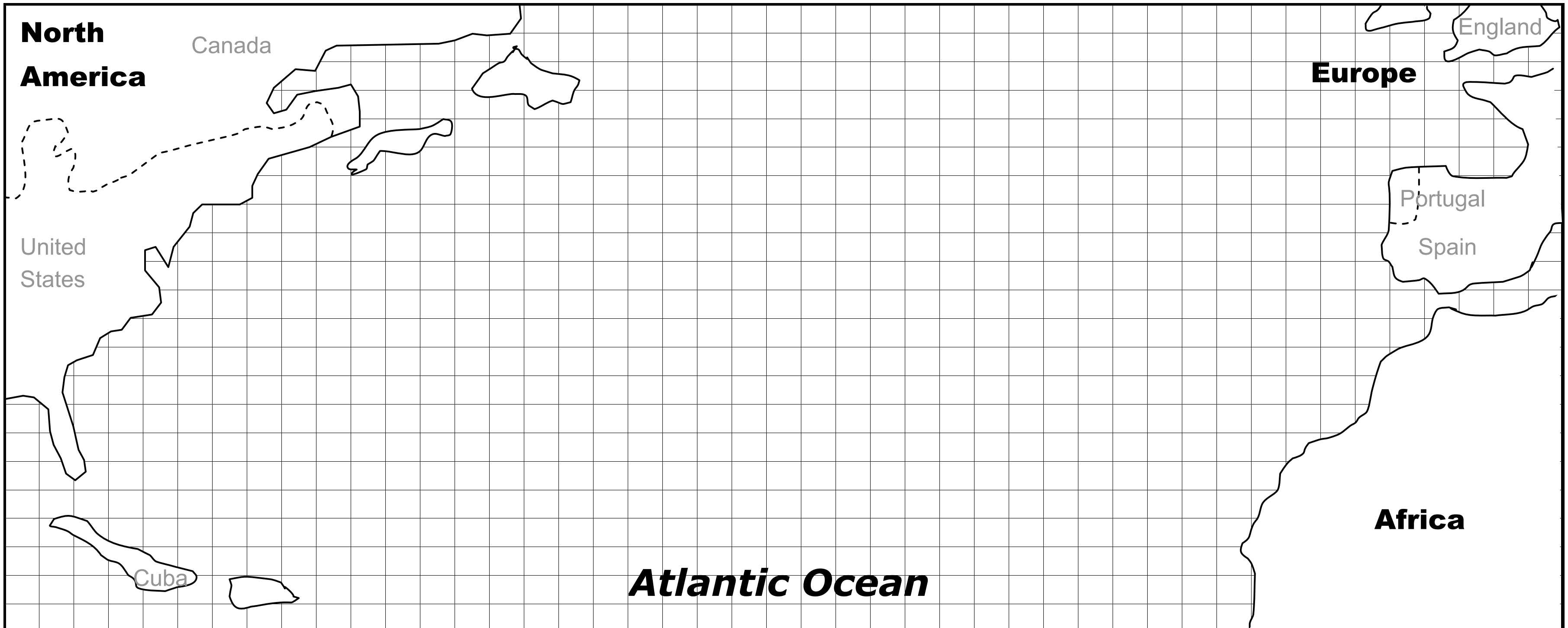
Ocean Vectors:

Month 1: 2 squares south Month 2: 4 squares west Month 3: 3 diagonal squares southwest

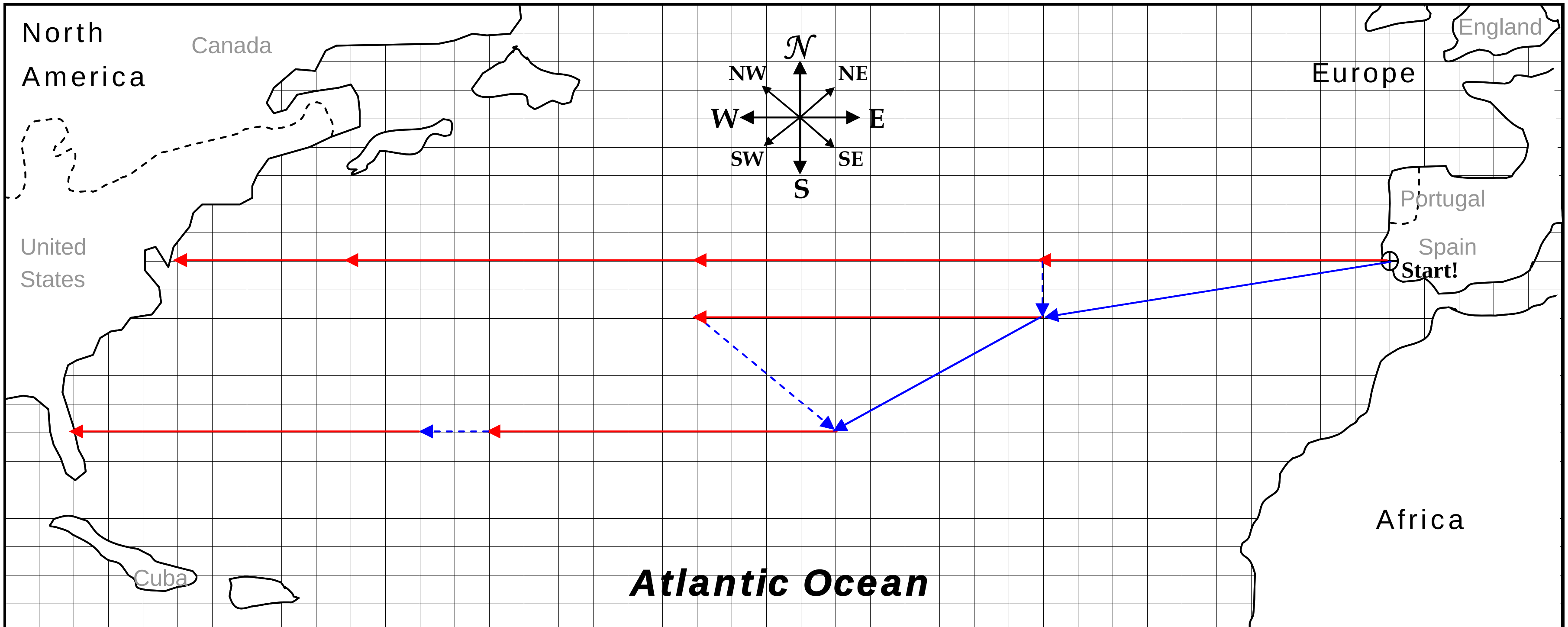
Where will you make landfall now? ___________ How many months to reach land (only count the solid 10-square vectors)?___________

Name: ______________________________ Date: ________________

Vector Voyage Worksheet



Vector Voyage *Solution* Worksheet 1



Vector Voyage Instructions

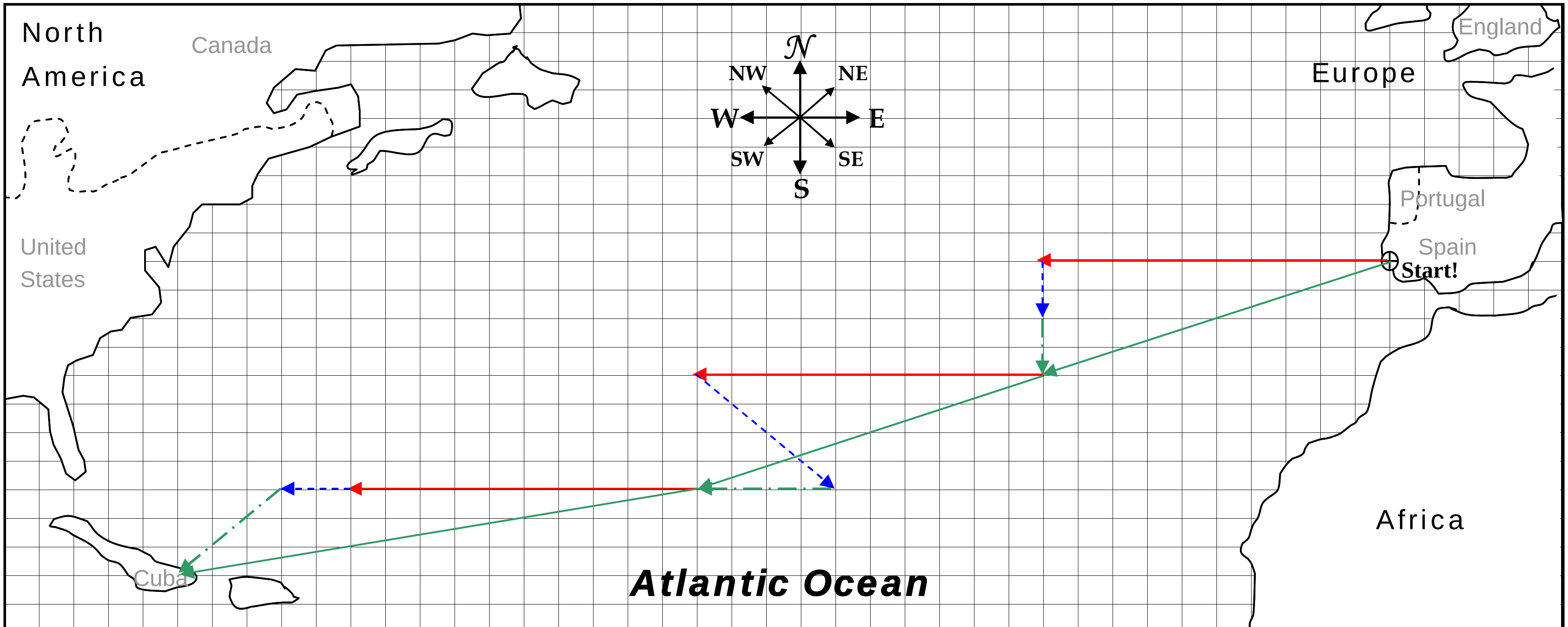
Part 1: Your ship can sail 10 squares/month. Sailing directly West from the Start position (Spain), draw one vector for each month of travel. In what country will you make landfall? **USA (NY)** How many months will it take to reach land (how many 10-square vectors is it)? **3.5**

Part 2: Unfortunately, the wind does not always blow the way you want! Each month, you must add a wind correction vector to find your position:

Month 1: 2 squares south Month 2: 4 diagonal squares southeast Month 3: 2 squares west Month 4: No wind

Where will you make landfall now? **USA (FL)** How many months to reach land (how many 10-square vectors is it)? **4**

Vector Voyage *Solution* Worksheet 2



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Wind Vectors:

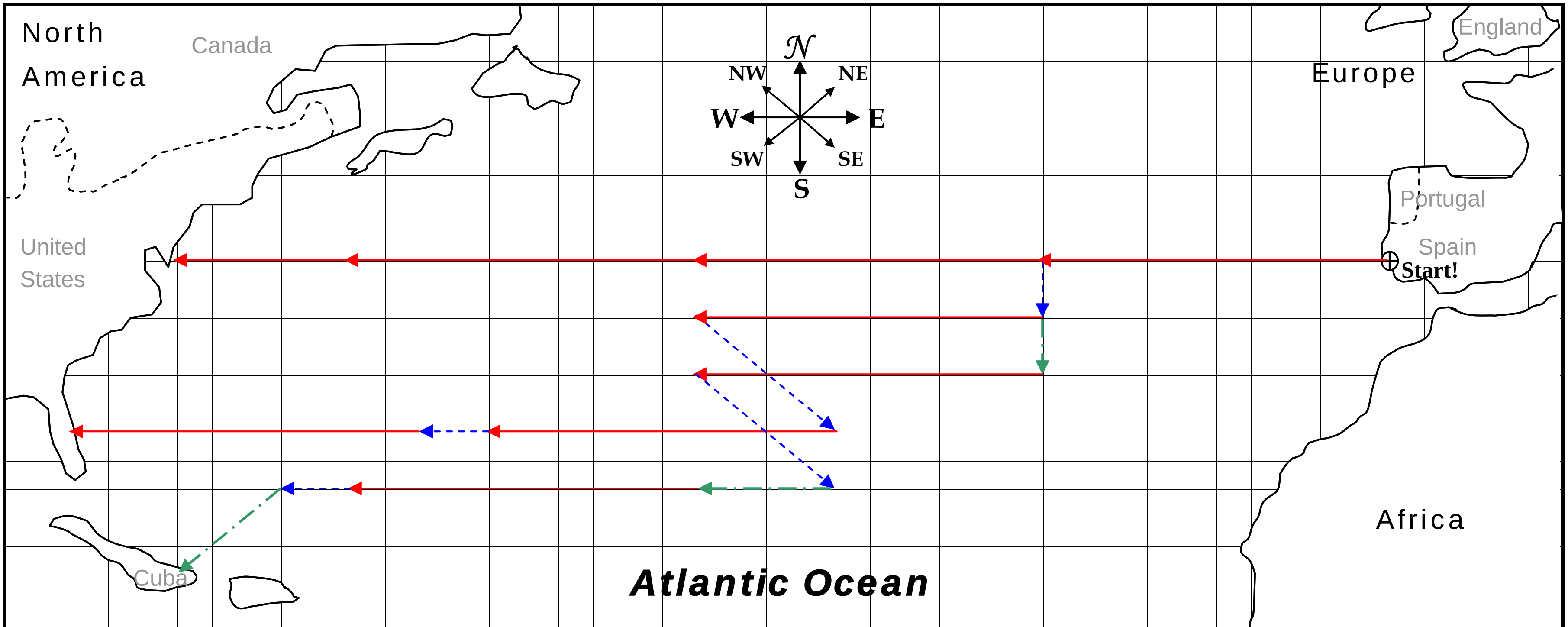
Month 1: 2 squares south Month 2: 4 diagonal squares southeast Month 3: 2 squares west Month 4: No wind

Ocean Vectors:

Month 1: 2 squares south Month 2: 4 squares west Month 3: 3 diagonal squares southwest

Where is your actual landfall? __**Cuba**__ How many months to actually reach land (how many 10 square vectors is it)? __3__

Vector Voyage *Solution* Worksheet 3



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